

# Algebra II

2011

Maryland Common Core State Curriculum Framework

Adapted from the Appendix A of the Common Core State Standards for Mathematics



DRAFT

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## Introduction

The Common Core State Standards for Mathematics (CCSSM) at the high school level specify the mathematics that all students should study in order to be college and career ready. The high school standards are listed in conceptual categories (number and quantity, algebra, functions, geometry, modeling, and probability and statistics). Consideration of how to organize the CCSSM high school standards into courses that provides a strong foundation for post secondary success was needed. To answer this charge, a group of experts, including state mathematics experts, teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives, were convened to develop Model Course Pathways in high school based on Common Core State Standards for Mathematics (CCSSM). The model pathways can be found in Appendix A of the Common Core State Standards for Mathematics.

After a review of these pathways, the superintendents of Maryland's LEA's voted to adopt the pathway reflected in this framework document which is referred to as the "Traditional Pathway". The "Traditional Pathway" consists of two algebra courses and a geometry course, with some data, probability and statistics included in each course.

## How to Read the Maryland Common Core Curriculum Framework for Algebra II

This framework document provides an overview of the standards that are grouped together to form the units of study for Algebra II. The standards within each unit are grouped by conceptual category and are in the same order as they appear in the Common Core State Standards for Mathematics. This document is not intended to convey the exact order in which the standards within a unit will be taught nor the length of time to devote to the study of the unit.

The framework contains the following:

- **Units** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards. A description of each cluster appears in the left column.
- **Cluster Notes** are instructional statements which relate to an entire cluster of standards. These notes are placed in the center column above all of the standards in the cluster.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must know and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard. The wording of some standards is so clear, however, that only partial support or no additional support seems necessary.
- **Standards** define what students should understand and be able to do.
- **Notes** are instructional notes that pertain to just one standard. They are placed in the center column immediately under the standard to which they apply. The notes provide constraints, extensions and connections that are important to the development of the standard.
- **Standards for Mathematical Practice** are listed in the right column.
- \* Denotes that the standard is a Modeling Standard. Modeling standards are woven throughout each conceptual category.
- (+) indicates additional mathematics that students should learn to prepare for advanced courses.

### Formatting Notes

- **Red Bold** - items unique to Maryland Common Core State Curriculum Frameworks
- **Blue bold** – words/phrases that are linked to clarifications
- **Black bold underline** - words within repeated standards that indicate the portion of the statement that is emphasized at this point in the curriculum or words that draw attention to an area of focus
- **Black bold** - Cluster Notes - notes that pertain to all of the standards within the cluster
- **Purple bold** – strong connection to current state curriculum for this course
- **Green bold** – standard codes from other courses that are referenced and are hot linked to a full description

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### **1. Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **2. Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**3. Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## 8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$  and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.



## Modeling Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

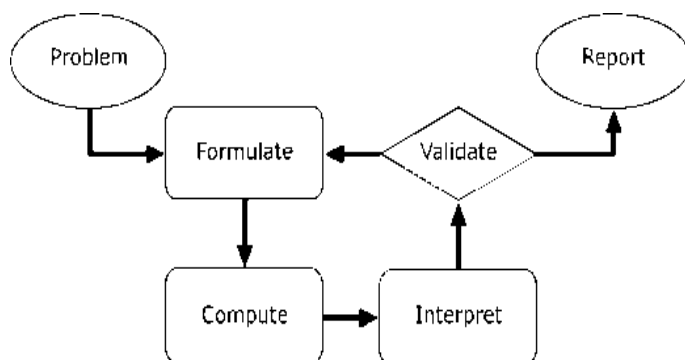
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram.



It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO<sub>2</sub> over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

**Modeling Standards** *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star ★.*

**(+) Standards** *The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.*

## Codes for Common Core State Standards (Math) Standards – K – 12

Grades K – 8		Applicable Grades
CC	Counting & Cardinality	K
EE	Expressions & Equations	6, 7, 8
F	Functions	8
G	Geometry	K, 1, 2, 3, 4, 5, 6, 7, 8
MD	Measurement & Data	K, 1, 2, 3, 4, 5
NBT	Number & Operations (Base Ten)	K, 1, 2, 3, 4, 5
NF	Number & Operations (Fractions)	3, 4, 5
NS	Number System	6, 7, 8
OA	Operations & Algebraic Thinking	K, 1, 2, 3, 4, 5
RP	Ratios & Proportional Relationship	6, 7
SP	Statistics & Probability	6, 7, 8
<b>Modeling</b>		
No Codes		Not determined
<b>High School</b>		
<b>Algebra (A)</b>		
A-APR	Arithmetic with Polynomial & Rational Expressions	8 -12
A-CED	Creating Equations	8 -12
A-REI	Reasoning with Equations & Inequalities	8 -12
A-SSE	Seeing Structure in Expressions	8 -12
<b>Functions (F)</b>		
F-BF	Building Functions	8 -12
F-IF	Interpreting Functions	8 -12
F-LE	Linear, Quadratic & Exponential Models	8 -12
F-TF	Trigonometric Functions	Not determined
<b>Geometry (G)</b>		
G-C	Circles	Not determined
G-CO	Congruence	Not determined
G-GMD	Geometric Measurement & Dimension	Not determined
G-MG	Modeling with Geometry	Not determined
G-GPE	Expressing Geometric Properties with Equations	Not determined
G-SRT	Similarity, Right Triangles & Trigonometry	Not determined
<b>Number &amp; Quantity (N)</b>		
N-CN	Complex Number System	Not determined
N-Q	Quantities	Not determined
N-RN	Real Number System	8 -12
N-VM	Vector & Matrix Quantities	Not determined
<b>Statistics (S)</b>		
S-ID	Interpreting Categorical & Quantitative Data	8 -12
S-IC	Making Inferences & Justifying Conclusions	Not determined
S-CP	Conditional Probability & Rules of Probability	Not determined
S-MD	Using Probability to Make Decisions	Not determined
<b>Modeling</b>		
No Codes		Not determined

## Algebra II

### Traditional Pathway

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. In this course rational functions are limited to those whose numerators are of degree at most one and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials

Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The critical areas for this course, organized into four units, are as follows:

**Critical Area 1:** This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

**Critical Area 2:** Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

**Critical Area 3:** In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as *“the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions”* is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

**Critical Area 4:** In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Units	Includes Standard Clusters*	Mathematical Practice Standards
<b>Unit 1</b> <b>Polynomial, Rational and Radical Relationships</b>	<ul style="list-style-type: none"> <li>Perform arithmetic operations with complex numbers.</li> <li>Use complex numbers in polynomial identities and equations.</li> <li>Interpret the structure of expressions.</li> <li>Write expressions in equivalent forms to solve problems.</li> <li>Perform arithmetic operations on polynomials.</li> <li>Understand the relationship between zeros and factors of polynomials.</li> <li>Use polynomial identities to solve problems.</li> <li>Rewrite rational expressions.</li> <li>Understand solving equations as a process of reasoning and explain the reasoning.</li> <li>Represent and solve equations and inequalities graphically.</li> <li>Analyze functions using different representations.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p>
<b>Unit 2</b> <b>Trigonometric Functions</b>	<ul style="list-style-type: none"> <li>Extend the domain of trigonometric functions using the unit circle.</li> <li>Model periodic phenomena with trigonometric function.</li> <li>Prove and apply trigonometric identities.</li> </ul>	<p><b>Use appropriate tools strategically.</b></p>
<b>Unit 3</b> <b>Modeling with Functions</b>	<ul style="list-style-type: none"> <li>Create equations that describe numbers or relationships.</li> <li>Interpret functions that arise in applications in terms of a context.</li> <li>Analyze functions using different representations.</li> <li>Build a function that models a relationship between two quantities.</li> <li>Build new functions from existing functions.</li> <li>Construct and compare linear, quadratic, and exponential models and solve problems.</li> </ul>	<p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
<b>Unit 4</b> <b>Inferences and Conclusions from Data</b>	<ul style="list-style-type: none"> <li>Summarize, represent, and interpret data on single count or measurement variable.</li> <li>Understand and evaluate random processes underlying statistical experiments.</li> <li>Make inferences and justify conclusions from sample surveys, experiments and observational studies.</li> <li>Use probability to evaluate outcomes of decisions.</li> </ul>	

\*In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

## Unit 1: Polynomial, Rational, and Radical Relationships

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Perform arithmetic operations with complex numbers.	<p>N.CN.1 Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to extend experience with solving quadratic equations with no real solution from Algebra I to the existence of complex numbers (e.g. use solving <math>x^2 + 1 = 0</math> as a way to introduce complex numbers)</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
	<p>N.CN.2 Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Knowledge of conjugate pairs and the nature of their products</li> </ul>	

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses

Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Use complex numbers in polynomial identities and equations.	<p>N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.</p> <p><b>Note:</b> Limit to polynomials with real coefficients.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to use the quadratic formula and/or completing the square as a means of solving a quadratic equation</li> <li>• Knowledge that complex solutions occur in conjugate pairs</li> <li>• Ability to connect experience with solving quadratic equations from Algebra I to situations where analyzing the discriminant will reveal the nature of the solutions which would include complex solutions</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
	<p>N.CN.8 (+) Extend <b>polynomial identities</b> to the complex numbers. <i>For example, rewrite</i>  <math>x^2 + 4</math> as <math>x^2 - (-4) = x^2 - (-2i)^2 = (x - (-2i))(x + (-2i)) = (x + 2i)(x - 2i)</math></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge that a negative number can be thought of as the square of an imaginary number  (e.g. <math>-4 = (-2i)^2</math>)</li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p>
	<p>N.CN.9 (+) Know the <b>Fundamental Theorem of Algebra</b>; show that it is true for quadratic polynomials.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge of the connection between the number of roots and the degree of the polynomial; considering multiple roots, complex roots and distinct real roots</li> </ul>	<p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

★ Modeling Standard

(+) indicate additional mathematics that students should learn in order to take advanced courses



Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Interpret the structure of expressions.	<p><b>Cluster Note:</b> Extend to polynomial and rational expressions.</p> <p>A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to connect experience in Algebra I with vocabulary that explicitly identifies coefficients, terms, and extend to degree, powers (positive and negative), leading coefficients, monomial... to more complicated expressions such as polynomial and rational expressions</li> <li>Ability to use appropriate vocabulary to categorize polynomials and rational expressions</li> </ul> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</p> <p><b>Note:</b> This is an overarching standard that has applications in multiple units and multiple courses.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>See the skills and knowledge that are stated in the Standard.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
	<p>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</p> <p><b>Note:</b> This is an overarching standard that has applications in multiple units and multiple courses.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to use properties of mathematics to alter the structure of an expression</li> <li>Ability to select and then use an appropriate factoring technique</li> <li>Ability to factor expressions completely over complex numbers</li> </ul>	

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Write expressions in equivalent forms to solve problems.	<p>A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example, calculate mortgage payments.</i> ★</p> <p><b>Note:</b> Consider extending this standard to infinite geometric series in curricular implementations of this course description.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge of the difference between an infinite and a finite series</li> <li>• Ability to apply the formula for the sum of a finite geometric series : <math>S_n = \frac{a(1-r^n)}{(1-r)}</math></li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Perform arithmetic operations on polynomials.	<p>A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are <b>closed</b> under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p> <p><b>Note: Extend to polynomials beyond quadratics</b></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experiences from Algebra I with linear and quadratic polynomials to polynomials of higher degree</li> <li>• Ability to show that when polynomials are added, subtracted or multiplied that the result is another polynomial</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Understand the relationship between zeros and factors of polynomials.	<p>A.APR.2 Know and apply the <b>Remainder Theorem</b>: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to make connections between factors, roots and evaluating functions</li> <li>• Ability to use both long division and synthetic division</li> <li>• Ability to use the graph of a polynomial to assist in the efficiency of the process for complicated cases</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
	<p>A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge of the differences in the end behavior of the graphs as dictated by the leading coefficient and whether the function is even or odd</li> <li>• Ability to capture the graphical behavior of polynomial functions which have roots with multiplicity greater than one</li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Use polynomial identities to solve problems.	<p><b>Cluster Note:</b> This cluster has many possibilities for optional enrichment, such as relating the example in A.APR.4 to the solution of the system <math>u^2 + v^2 = 1</math>, <math>v = t(u+1)</math>, relating the Pascal triangle property of binomial coefficients to <math>(x+y)^{n+1} = (x+y)(x+y)^n</math>, deriving explicit formulas for the coefficients, or proving the binomial theorem by <b>induction</b>.</p> <p>A.APR.4 Prove <b>polynomial identities</b> and use them to describe numerical relationships. For example, the polynomial identity <math>(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2</math> can be used to generate Pythagorean triples.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge of the process for proving identities</li> <li>• Ability to see, use and manipulate the structure in an expression as needed to prove an identity</li> </ul>	<p>Make sense of problems and persevere in solving them.</p> <p>Reason abstractly and quantitatively.</p> <p>Construct viable arguments and critique the reasoning of others.</p> <p>Model with mathematics.</p> <p>Use appropriate tools strategically.</p> <p>Attend to precision.</p> <p>Look for and make use of structure.</p> <p>Look for and express regularity in repeated reasoning.</p>
	<p>A.APR.5 (+) Know and apply the <b>Binomial Theorem</b> for the expansion of <math>(x + y)^n</math> in powers of <math>x</math> and <math>y</math> for a positive integer <math>n</math>, where <math>x</math> and <math>y</math> are any numbers, with coefficients determined for example by Pascal's Triangle.</p> <p><b>Note:</b> The <b>Binomial Theorem</b> can be proved by <b>mathematical induction</b> or by combinatorial argument.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to replicate Pascal's triangle</li> </ul>	

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Rewrite rational expressions	<p>A.APR.6 Rewrite simple rational expressions in different forms; write <math>\frac{a(x)}{b(x)}</math> in the form <math>q(x) + \frac{r(x)}{b(x)}</math> where <math>a(x), b(x), q(x)</math> and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p><b>Note:</b> The limitations on rational functions apply to the rational expressions in this standard. <i>Limitations: In this course rational functions are limited to those whose numerators are of degree at most one and denominators of degree at most 2.</i></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to make connections to the Remainder Theorem</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
	<p>A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, <b>closed</b> under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p><b>Note:</b> A.APR.7 requires the <b>general division algorithm</b> for polynomials.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to make connections between the algorithms for operations on rational numbers and operations on rational expressions</li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Understand solving equations as a process of reasoning and explain the reasoning.	<p>A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect prior experience with solving simple equations in one variable to solving equations which require new strategies and additional steps</li> <li>• Ability to make connections between the domain of a function and extraneous solutions</li> <li>• Ability to identify extraneous solutions</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Represent and solve equations and inequalities graphically.	<p>A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find <b>successive approximations</b>. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. <sup>★</sup></p> <p><b>Note:</b> Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.</p> <p><b>Note:</b> This is an overarching standard that will be revisited as each function is studied.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with solving systems of equations graphically from Algebra I to solving systems that include polynomial, exponential, rational, root, absolute value and logarithmic functions</li> <li>• Ability to show the equality of two functions using multiple representations</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 1: Polynomial, Rational, and Radical Relationships		
Cluster	Standard	Practices
Analyze functions using different representations.	<p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <sup>★</sup></p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p><b>Note:</b> Relate this standard to the relationship between zeros of quadratic functions and their factored forms.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with graphing linear, exponential and quadratic functions from Algebra I to graphing polynomial functions</li> <li>• Ability to identify key features of a function: max, min, intercepts, zeros, and end behaviors.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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## Unit 2: Trigonometric Functions

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Unit 2: Trigonometric Functions		
Cluster	Standard	Practices
Extend the domain of trigonometric functions using the unit circle.	<p>F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge that angle measures in radians may be determined by a ratio of intercepted arc to radius</li> <li>• Ability to convert between degree and radian measure</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
	<p>F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect knowledge of special right triangles gained in Geometry to evaluating trigonometric functions at any domain value</li> <li>• Ability to extend to angles beyond <math>[-2\pi, 2\pi]</math>, using counterclockwise as the positive direction of rotation</li> </ul>	

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Unit 2: Trigonometric Functions		
Cluster	Standard	Practices
Model periodic phenomena with trigonometric functions.	<p>F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to connect contextual situations to appropriate trigonometric function: e.g. using sine or cosine to model cyclical behavior</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 2: Trigonometric Functions		
Cluster	Standard	Practices
Prove and apply trigonometric identities.	<p>F.TF.8 Prove the <b>Pythagorean identity</b> <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math>, given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math>, and the quadrant of the angle.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to make connections to angles in standard position</li> </ul> <p><b>Note:</b> An Algebra II course with an additional focus on trigonometry could include the (+) standard F.TF.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. This could be limited to acute angles in Algebra II.</p>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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### Unit 3: Modeling with Functions

In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. *The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.*

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Create equations that describe numbers or relationships.	<p>Cluster Note: For A.CED.1, use all available types of functions to create such equations, including <b>root functions</b>, but constrain to simple cases. While functions used in A.CED.2, 3, and 4 will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.</p> <p><i>Limitations: In this course rational functions are limited to those whose numerators are of degree at most one and denominators of degree at most 2.</i></p> <p>A.CED.1 Create equations and inequalities in <b>one</b> variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and <b>simple rational</b> and exponential functions.</i></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience from Algebra I with creating linear, exponential and quadratic equations in one variable to include creating <b>simple rational functions</b></li> <li>• Ability to distinguish between linear, quadratic, exponential, root and <b>simple rational relationships</b> given the verbal, numeric and/or graphic representations</li> </ul>	<p>Make sense of problems and persevere in solving them.</p> <p>Reason abstractly and quantitatively.</p> <p>Construct viable arguments and critique the reasoning of others.</p> <p>Model with mathematics.</p> <p>Use appropriate tools strategically.</p> <p>Attend to precision.</p> <p>Look for and make use of structure.</p> <p>Look for and express regularity in repeated reasoning.</p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
<p>Create equations that describe numbers or relationships.</p> <p>(continued from previous page)</p>	<p>A.CED.2 Create equations in <b>two or more</b> variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p><b>Note: These are overarching standards that will be revisited for each function studied but as each new function is introduced, modeling problems should not be limited to just the newly introduced function but should include all functions studied to date including those studied in Algebra I.</b></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to distinguish between linear, quadratic exponential and simple rational relationships given multiple representations</li> <li>• Ability to determine unknown parameters needed to create an equation that accurately models a given situation</li> <li>• Ability to produce the graph of a function, that is being used to model relationships, which would include appropriate scales and labels as to accurate display all aspects of the relation</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
	<p>A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to distinguish between a mathematical solution and a contextual solution</li> </ul> <p>A.CED.4 Rearrange formulas to highlight a <b>quantity of interest</b>, using the same reasoning as in solving equations.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with rearranging formulas that were linear in nature from Algebra I to manipulating formulas that are not linear in nature</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Interpret functions that arise in applications in terms of a context.	<p><b>Cluster Note:</b> Emphasize the selection of a model function based on behavior of data and context.</p> <p><b>Note:</b> These are overarching standards that will be revisited for each function studied but as each new function is introduced, modeling problems should not be limited to just the newly introduced function but should include all functions studied to date including those studied in Algebra I.</p> <p>F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> ★</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect appropriate function to context</li> <li>• Knowledge of the key features of linear, exponential, polynomial, root, absolute value, piece-wise, simple rational, logarithmic and trigonometric functions</li> </ul> <p>F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i> ★</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to relate the concept of domain to each function studied</li> <li>• Ability to describe the restrictions on the domain of all functions studied based on real world context</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
<p>Interpret functions that arise in applications in terms of a context</p> <p>(continued from the previous page)</p>	<p>F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>Ability to apply this skill to linear, quadratic, polynomial, root and simple rational functions</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Analyze functions using different representations.	<p><b>Cluster Note:</b> Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.</p> <p>F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with graphing linear and quadratic functions from Algebra I to graphing square root, cube root and a variety of piece-wise defined functions</li> <li>• Ability to produce a rough graph of the parent function for each type of function</li> <li>• Knowledge of how parameters introduced into a function alter the shape of the graph of the parent function</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p>
	<p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with graphing linear and quadratic functions from Algebra I to graphing exponential and logarithmic functions</li> <li>• Ability to produce a rough graph of the parent function for each type of function</li> <li>• Knowledge of how parameters introduced into a function alter the shape of the graph of the parent function</li> </ul>	<p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
<b>Analyze functions using different representations.</b>  (continued from the previous page)	F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  <b>Essential Skills and Knowledge</b> <ul style="list-style-type: none"> <li>• Ability to connect experience with writing linear, quadratic and exponential functions in various forms from Algebra I to writing all functions in various forms</li> <li>• Ability to recognize functions in various forms</li> </ul>	<b>Make sense of problems and persevere in solving them.</b>  <b>Reason abstractly and quantitatively.</b>  <b>Construct viable arguments and critique the reasoning of others.</b>
	F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>  <b>Essential Skills and Knowledge</b> <ul style="list-style-type: none"> <li>• Ability to recognize common attributes of functions from various representations</li> </ul>	<b>Model with mathematics.</b>  <b>Use appropriate tools strategically.</b>  <b>Attend to precision.</b>  <b>Look for and make use of structure.</b>  <b>Look for and express regularity in repeated reasoning.</b>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Build a function that models a relationship between two quantities.	<p>F.BF.1 Write a function that describes a relationship between two quantities. <sup>★</sup></p> <p>b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p> <p><b>Note:</b> Develop models for more complex or sophisticated situations than in previous courses.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with adding, subtracting, multiplying and dividing linear, quadratic and exponential functions from Algebra I to adding, subtracting, multiplying and dividing any functions</li> <li>• Ability to create a new function that is the composition of two or more functions</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Build new functions from existing functions.	<p>F.BF.3 Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing <b>even and odd functions</b> from their graphs and algebraic expressions for them.</p> <p><b>Note:</b> Use transformations of functions to find models as students consider increasingly more complex situations. Note the effect of multiple transformations on a single graph and the common effect of each transformation across function types.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with this standard as it relates to linear, quadratic and exponential functions from Algebra I to all functions studied</li> <li>• Ability to make generalizations about the changes that will take place in the graph of any function as a result of making a particular change to the algebraic representation of the function</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p>
	<p>F.BF.4 Find inverse functions.</p> <p>a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></p> <p><b>Note:</b> Extend this standard to <b>simple rational</b>, simple radical, and simple exponential functions; connect this standard to F.LE.4. <b>i.e. use inverses to show the connection between exponential and logarithmic functions.</b></p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to connect experience with finding the inverse of a linear function from Algebra I to finding the inverse of simple exponential, root and rational functions</li> <li>• Knowledge of the connection of the domain and range of a function to its inverse</li> <li>• Ability to determine if a function has an inverse</li> </ul>	<p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 3: Modeling with Functions		
Cluster	Standard	Practices
Construct and compare linear, quadratic, and exponential models and solve problems.	<p>F.LE.4 For exponential models, express as a logarithm the solution to a <math>b^{ct} = d</math> where <math>a</math>, <math>c</math>, and <math>d</math> are numbers and the base <math>b</math> is 2, 10, or <math>e</math>; evaluate the logarithm using technology.</p> <p><b>Note:</b> Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that <math>\log xy = \log x + \log y</math>.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge that logarithmic functions are inverses of exponential functions</li> <li>• Knowledge of the properties of logarithms and exponents and their connection to one another</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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### Unit 4: Inferences and Conclusions from Data

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
Summarize, represent, and interpret data on a single count or measurement variable.	<p>S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p> <p><b>Note:</b> While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to construct, interpret and use normal curves, based on standard deviation</li> <li>• Ability to identify data sets as approximately normal or not</li> <li>• Ability to estimate and interpret area under curves using the Empirical Rule (68-95-98.7%)</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
Understand and evaluate random processes underlying statistical experiments.	<p>S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Knowledge of various sampling methods (e.g., simple random, convenience, stratified...)</li> <li>• Ability to select an appropriate sampling technique for a given situation</li> <li>• Ability to explain in context the difference between values describing a population and a sample</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
	<p>S.IC.2 Decide if a specified model is consistent with results from a given <b>data-generating process</b>, e.g., using simulation. <i>For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</i></p> <p><b>Note:</b> Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to calculate and analyze theoretical and experimental probabilities accurately</li> <li>• Knowledge of various types of sampling procedures and ability to select and carry out the appropriate process for a given situation</li> <li>• Ability to design, conduct and interpret the results of simulations</li> <li>• Ability to explain and use the <b>Law of Large Numbers</b></li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
<p><b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b></p>	<p>S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</p> <p><b>Note:</b> In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to conduct sample surveys, experiments and observational studies</li> <li>• Understanding of the limitations of observational studies that do not allow major conclusions on treatments</li> <li>• Ability to recognize and avoid bias</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
<p><b>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</b></p> <p>(continued from the previous page)</p>	<p>S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</p> <p><b>Note:</b> For S.IC.4 and 5, focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to informally establish bounds as to when something is statistically significant</li> <li>• Ability to conduct simulations and accurately interpret and use the results</li> <li>• Ability to use sample means and sample proportions to estimate population values</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
<p>Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</p> <p>(continued from the previous page)</p>	<p>S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to set up and conduct a randomized experiment or investigation, collect data and interpret the results</li> <li>• Ability to draw conclusions based on comparisons of simulation versus experimental results</li> <li>• Ability to determine the statistical significance of data</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p>
	<p>S.IC.6 Evaluate reports based on data.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• See the skills and knowledge that are stated in the Standard.</li> </ul>	<p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>

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Unit 4: Inferences and Conclusions from Data		
Cluster	Standard	Practices
Use probability to evaluate outcomes of decisions.	<p>S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p> <p><b>Note:</b> Extend to more complex <b>probability models</b>. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• See the skills and knowledge that are stated in the Standard.</li> </ul>	<p><b>Make sense of problems and persevere in solving them.</b></p> <p><b>Reason abstractly and quantitatively.</b></p> <p><b>Construct viable arguments and critique the reasoning of others.</b></p> <p><b>Model with mathematics.</b></p> <p><b>Use appropriate tools strategically.</b></p> <p><b>Attend to precision.</b></p> <p><b>Look for and make use of structure.</b></p> <p><b>Look for and express regularity in repeated reasoning.</b></p>
	<p>S.MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p> <p><b>Essential Skills and Knowledge</b></p> <ul style="list-style-type: none"> <li>• Ability to synthesize and apply various probability concepts to evaluate decisions</li> </ul>	

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